



Dynamic Analysis of Vehicle-Structure Interaction in Indian Railways BOXN25 Wagon-Steel Plate Girder Bridge under Semi High Speed and Heavy Axle Load

Mohammad Islam*

Synopsis:

This study presents the dynamic analysis of vehicle-structure interaction in BOXN25 wagon-steel plate Girder Bridge under semi high speed and heavy axle load. Indian railways BOXN25 wagon (mounted on CASNUB-22 HS) bogie considered as a vehicle and Indian railways steel plate Girder bridge considered as a Bridge structure as per drawing "RDSO/B-16005", A 10 DOFs mathematical model of the wagon and Bridge structure is designed for this study. This is interaction problem has been solved numerically with help of condensation technique developed by Yang and Wu (2001). In this study, a series of two locos and four BOXN25 wagons are considered for dynamic response of bridge with heavy axle load and semi high speed (case no 2) as per load combination 1, ("25t loading -2008" bridge rule) as a moving force model, dynamic response of bridge and vehicle are also observed under a series of four BOXN 25 wagon with duly taken in to account the inertia force and suspension system of wagon (case no 3). A parametric study also performed for dynamic responses of the vehicle and bridge under different speed of the vehicle. Design aspect of the bridge as per drawing "RDSO/B-16005" also checked with increased axle load, CDA and DAF for bridge are calculated with different speed of vehicle under this study

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are compared with CDA value calculated for same span as per bridge rule 2008.

Key words: Vehicle, Bridge, interaction, Dynamic Response, Modeling, Suspension system, CDA

1. Introduction:

The vibration of bridges caused by the passing of vehicles has been a subject of research since nineteenth century and the research has been accelerated in this discipline after introducing the high speed train. Lot of research work has been done by many researchers considering different type of model for vehicle and bridge; vehicles were often approximated as moving loads, which in many cases allows the problem to be solved analytically. Extensive references to the literature on the subject can be found in the book by Fryba([1-2]), [3-4]. Moving loads model applicable only when vehicle to bridge mass ratio is small. By taken into the consideration of inertia effect of moving vehicles, the moving mass model has been adopted instead of moving loads [5-6]. Nevertheless, it does not allow consideration of the interaction and bouncing action of the moving vehicle relative to the bridge. Such an effect is expected to be significant for vehicles moving at rather high speeds. By taken into an account of interaction of the moving mass with bridge, unsprung mass model has been considered [7-8]. For the cases where only the bridge response is desired, the moving vehicles have frequently been approximated to the extreme as a number of moving loads and moving masses. However, for case where the riding comfort or vehicle response is of concern, it is necessary to consider the effect of the suspension systems and the interaction of vehicle with bridge. Owing to the construction of high-speed railways and the upgrading of existing railways world-wide, the problem of vehicle–bridge interactions has increased the attention of scientific fraternity of the world, the simplest model in this regard is a lumped mass supported by a spring–dashpot unit, often referred to as the sprung mass model [9-11], though more sophisticated



models can be devised for the vehicles such as rigid suspended bar supported by spring–dashpot unit with multi wheel model of the vehicle [12-15]

The main objective of my work is to design a model for Indian Railway freight vehicle, which can adequately account for the inertia, dynamic properties of the moving vehicle and also interaction of the vehicle with bridge structure, and a model of Indian railway plate Girder Bridge for dynamic responses of Railway Bridge and vehicle. To solve this interaction problem, a condensation technique has been adopted, which was developed by Yang and Wu (2001), [14]. This interaction problem consist of two sets of equations of motion of the second order, must be written each for the vehicle and for the bridge. It is the interaction forces existing at the contact points that make the two subsystems coupled. As the contact points move from time to time, the system matrices are time-dependent and must be updated and factorized at each time step in an incremental analysis. At initial stage of this work number of simple numerical problem are considered such as moving single force, single sprung mass with 2DOFs, two wheel sprung mass model with total of 4 DOFs and finally four wheel vehicle model with total of 10 DOFs, these problems numerically are solved by FEM in MATLAB for different parameters of vehicle and bridge from available literatures, and results obtained are confirmed same. Eventually a series of BOXN25 and other Wagons having axle load caring capacity 25 ton are considered to develop mathematical model for dynamic analysis

2.0 MATHEMATICAL MODEL OF VEHICLE BRIDGE STRUCTURE.

2.1 MODELING OF VEHICLE

The simplest model of the vehicle in this context is moving force model, for more complex model of the vehicle, the wagon body is modeled as a rigid body having a total mass M_{ct} ; and mass moment of inertia I_c about the transverse horizontal centroidal axes. Similarly, each bogie frame is considered as a rigid body with a total

mass M_{tt} (M_{tt1} and M_{tt2}) and mass moment of inertia I_t about the transverse horizontal centroidal axes. Each axle along with the two wheel set has a total mass M_{wt} (for four axles M_{wt1} ; M_{wt2} ; M_{wt3} and M_{wt4}). The spring and the shock absorber in the primary suspension for each axle are characterized by spring stiffness K_p and damping constant C_p respectively. Likewise, the secondary suspension is characterized by spring stiffness k_s and damping constant c_s respectively. As the vehicle wagon body is assumed to be rigid, its motion may be described by the vertical displacement (bounce or v_e) and rotations about the transverse horizontal axis e (pitch). Similarly the movements of the two bogie units are described by two degrees of freedom (vertical translation and rotation) V_{bf} , b_f and V_{br} , b_r each about their centroids. The vehicle model have total number of four axles, each axle is described by one vertical degrees of freedom v_{wi} , totally this vehicle model compromises of 10 degrees of freedom (Fig.2.2). The detailed parameter regarding the mass of different component for the vehicle is tabulated in table 3.2. Parameter of primary and secondary suspension system of vehicle is tabulated in table. The two-dimensional view of a railway BOXN25 vehicle moving over a railway bridge with a constant speed v on straight track is presented in the Fig.2.2. The primary and secondary suspension systems of the vehicle are represented by a



Fig. 2.1: Indian Railway BRNA flat wagon

b_r , each about their centroids. The vehicle model have total number of four axles, each axle is described by one vertical degrees of freedom v_{wi} , totally this vehicle model compromises of 10 degrees of freedom (Fig.2.2). The detailed parameter regarding the mass of different component for the vehicle is tabulated in table 3.2. Parameter of primary and secondary suspension system of vehicle is tabulated in table. The two-dimensional view of a railway BOXN25 vehicle moving over a railway bridge with a constant speed v on straight track is presented in the Fig.2.2. The primary and secondary suspension systems of the vehicle are represented by a

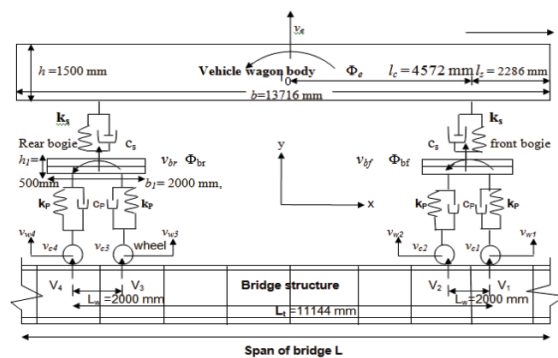


Fig. 2.2: Two dimensional view of the vehicle model used in this study



spring-dashpot unit. All the wheels are assumed to be in close contact with the railway bridge as they roll over.

2.1.1 Assumption in modeling of railway vehicle

- 1) The wagon body, bogies and axle-sets in wagon are regarded as rigid components,
- 2) The connections between a bogie and its wheel-sets are characterized by the first suspension system and the connections between a wagon body and its bogies are characterized by the second suspension system.
- 3) The springs and dampers of the primary and secondary suspension system elements have linear characteristics and damping in both the suspension system is velocity proportional.
- 4) Friction does not exist between axle and bearing.
- 5) The vehicle runs at a constant speed on a straight Bridge.
- 6) All wheels remain in contact with the bridge surface; there is no bouncing between bridge structure and wheels.
- 7) The friction force acting between the wheel and bridge contact surface and horizontal contact point forces have been neglected.
- 8) Coriolis force effect induced on the bridge by moving vehicle has been neglected

2.2 MODELING OF BRIDGE

A single span, prismatic I- section plate girder Railway Bridge as per drawing "RDSO/B-16005 considered to analyze the dynamic response of railway Bridge under the moving vehicle (BOXN25 wagon mounted on CASNUB-22 HS bogie). The behavior of single span, prismatic I- section plate girder Bridge is described by the Euler Bernoulli beam theory, for the purpose of FE analysis, the span of bridge discretized in to ten no of equal length finite beam element with total 20 DOFs, each beam element are represented by two vertical translation DOFs and two rotational DOFs, total of four DOFs. The response of beam element in between the nodal point is characterize by the hermetian interpolation shape function, the discretize bridge and beam element of the bridge with DOFs are

shown in the Fig.2.4, and properties of bridge are summarized in table 3.3



Fig.2.3: Physical view of the plate girder railway bridge

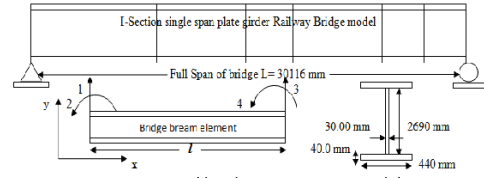


Fig.2.4: Span of bridge, X-section and finite beam element of bridge

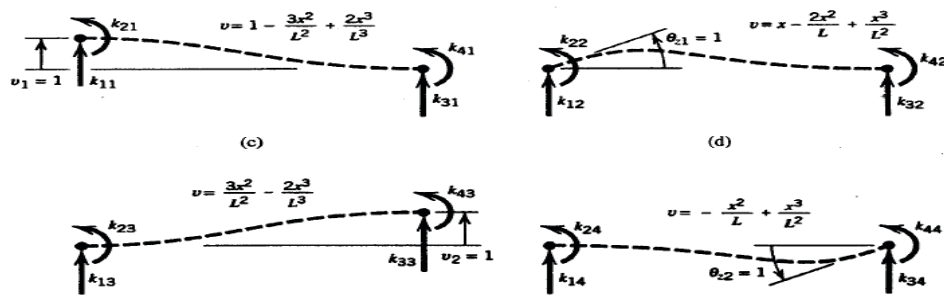


Fig.2.5: Shape function and elastic deform line for bridge beam element

2.3 Assumption in modeling of Railway Bridge

- 1) Mass per unit length of the bridge is constant.
- 2) Cross section area of bridge along the span of bridge is constant.
- 3) Small deformation theory is considered,
- 4) The damping of bridge sub-structure is considered as Rayleigh's damping,
- 5) The computation will be carried through for a simple supported beam.
- 6) For FE analysis, the complete span of bridge is divided in equal length of beam elements.
- 7) Straight Railway Bridge is considered for this study

RESULTS

3.0 Introduction

The main rationale of this project was to perform study on the dynamic responses of the Bridge and vehicle, with vehicle characteristics, which is moving over the bridge with accounting the interaction of the vehicle structure to bridge structure, the



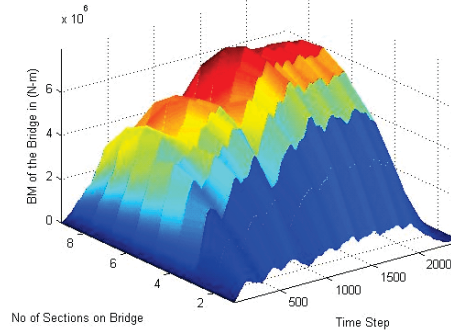
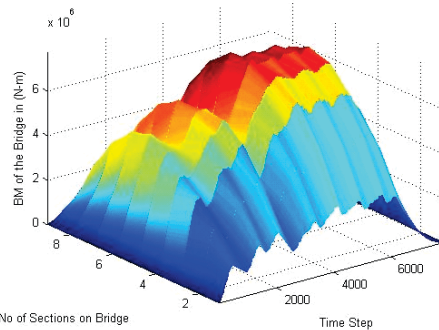
mathematical model of the vehicle and bridge already has been described in fig 2.2 & 2.4 rep. In order to solve this complex problem, MATLAB is used. This Vehicle Bridge Interaction problem solved numerically by FE method, Newmark's finite difference scheme is used to discretize the vehicle equations of motion. Dynamic responses of the Bridge under moving vehicles has been determined and presented below. Total of three cases are considered in this study.

3.1 CASE NO. 1: Static analysis

In case no. 1, a simulation of two locos and four BOXN25 wagons 25 T axle load rake as per load combination 1, ("25t loading -2008" bridge rule) are placed on bridge at different positions for midpoint displacement and BM, and static displacement and BM for mid sections at different position of first axle of first loco with respect to left end of bridge are presented in fig. 3.1 & 3.2, respectively with solid line. It is clear from response that maximum displacement and bending moment at mid sections occur when there is rear bogie of second loco and wagons are occupy the bridge.

3.2 CASE NO. 2: Dynamic analysis with moving force model

In case no.2, a simulation of moving load model of two locos and four BOXN25 wagons 25 T axle load rake as per load combination 1, ("25t loading -2008" bridge rule) are passed with a constant speed of 160 kmph over bridge. Displacement and BM for mid sections at different position of first axle of first loco with respect to left end of bridge are presented in fig. 3.1 & 3.2, respectively with dotted line. It is clear from the responses that maximum displacement and bending moment at mid sections occur when there is rear bogie of second loco and wagons are occupy the bridge.



BM Envelopes of bridge under the action of static and moving load of “load combination 1 as per bridge rule 2008” two locos and four wagons respectively.

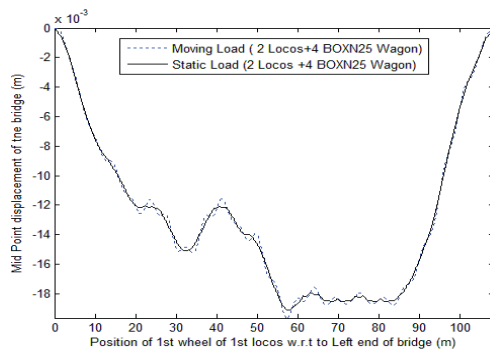


Fig. 3.1: Midpoint Displacement

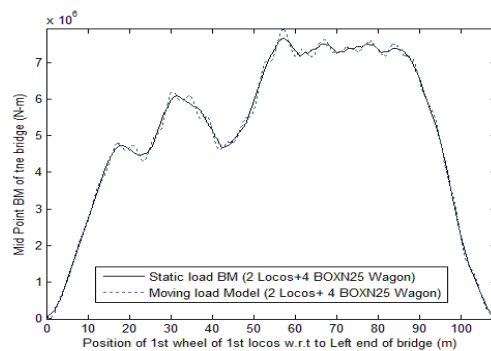


Fig. 3.2: Midpoint BM

Table no.: 3.1 Displacement and BM under static and moving load model

Live Load	Displacement (mm)		Bending Moment(N-m)	
	EUDL	Actual analysis	EUDL	Actual analysis
Static condition	19.92	19.1	8.121×10^6	7.72×10^6
Moving load		19.7		7.92×10^6

3.3 CASE NO. 3: Dynamic analysis with vehicle model

In present study a series of four BOXN25 wagon type of rolling stock are selected, mathematical models of that rolling stock is developed in 2.1, and that series of vehicles are passed from a single span steel plate girder bridge which is a mathematical model developed also in 2.2, with a constant speed of 160 kmph. This vehicles bridge interaction problem is solved with help of condensation technique developed by Yang and Wu in 2001 in



MATLAB specific data for vehicles and bridge models are available in table 3.2 & 3.3 respectively. Results for BM envelop, midpoint displacement, vertical acceleration and BM of the bridge under moving vehicle, moving load and static load condition are presented in fig 3.3, 3.4 3.5 & 3.6 respectively. Vertical acceleration of the first wagon body is presented in fig. 3.7, Effect of vehicles speed on midpoint displacement and bending moment are presented in fig. 3.8 & 3.9 rply. Wagon sensitivity is presented in fig. 3.10 & midpoint max. BM in table no 3.3.1

TABLE 3.2: Parameter of BOXN25 Wagon with CASNUB-22 HS bogie (Wagon maintenance Manual)

Parameters	Unit	Empty	Loaded
Mass of wagon body	t	12.144	89.144
Mass of bogie	t	2.288	2.288
Mass of axle	t	1.6	1.6
Mass moment of inertia of car body	Kg-m ²	111000	701193.67
Mass moment of inertia of bogie	Kg-m ²	483	483
Stiffness of primary suspension system per axle	N/m	98100000	98100000
Damping of primary suspension system per axle	Ns/m	10000*	10000*
Stiffness of secondary suspension system per bogie	N/m	8835000	8835000
Damping of secondary suspension system per bogie	Ns/m	7165	7165
Center to center distance between two wagon	m	10.713	10.713
Center to center distance between two axle	m	2.00	2.00

* assumed

3.4 Properties of steel plate Girder Bridge

The properties of steel plate girder are taken from IR drawing nos. RDSO/B-16005 and listed in table no. 3.3

TABLE 3.3: Parameters for Steel Plate Girder Railways Bridge Model [RDSO, B/16005]

S. No	Parameter	Symbol	Value
1	Young 's modulus	E	$210 \times 10^9 \text{ N/m}^2$
2	Moment of inertia of bridge	I_b	0.1325 m^4
3	Mass per unit length of the bridge	m_b	1730 kg/m
4	Fundamental frequency of the bridge structure	f_1	9.6 Hz
5	Span of bridge	L_c	24.4 m
6	Overall span	L_o	26.050 m
7	Effective span	L	25.6 m

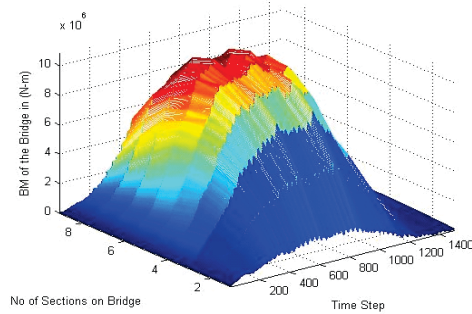


Fig 3.3 BM Envelope under the action of BOXN 25

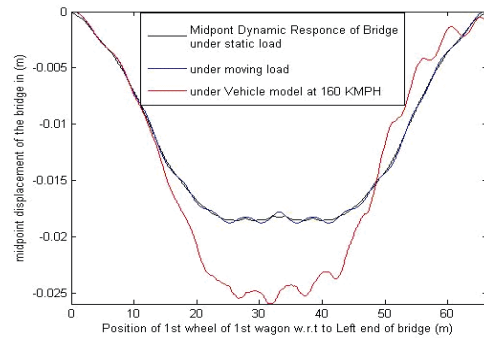


Fig. 3.4 Midpoint displacement of bridge

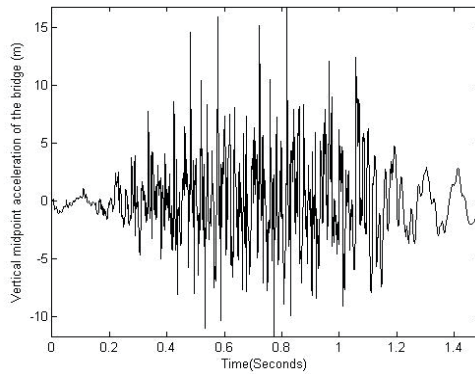


Fig. 3.5 Midpoint acceleration of bridge

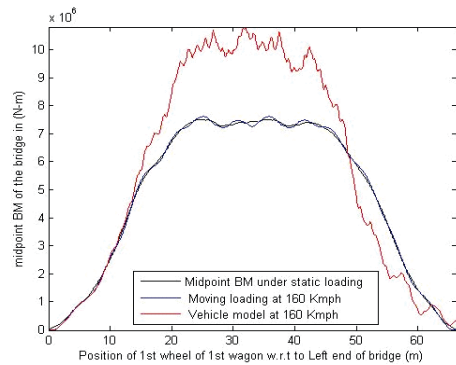


Fig. 3.4 Midpoint displacement of bridge

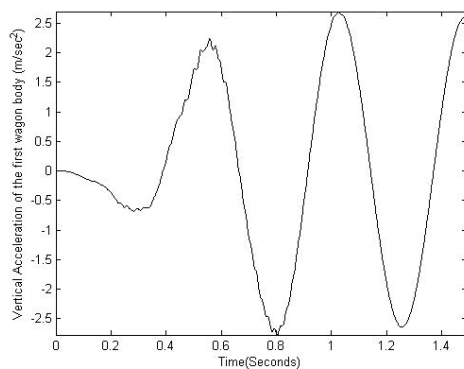


Fig. 3.7 Vertical Acc. of 1st wagon body

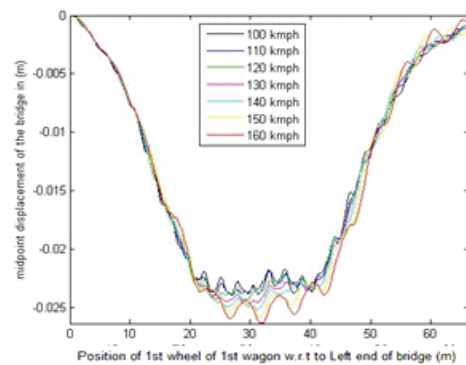


Fig.3.8 Midpoint Displacement of Bridge

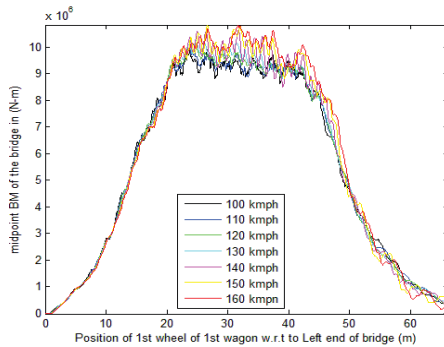


Fig.3.9 Midpoint BM of Bridge

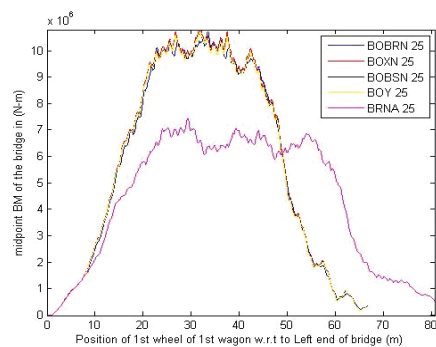


Fig.3.10 Wagon sensitivity to the midpoint BM

Table 3.3.1 Maximum BM at the centre of bridge under different wagon with same axle load

Wagon type	BOXN25	BOBRN25	BOBSN25	BOY25	BRNA25
Max. BM (N-m)	10.81 x10⁶	10.72 x10 ⁶	10.68 x10 ⁶	10.68 x10 ⁶	7.43 x10 ⁶

Table 3.4: CDA and DAF for 24.4 m steel plate Girder Bridge.

Speed of the Rolling stock	Midpoint Displacement	Midpoint BM	CDA % static BM(7.51x10 ⁶)	DAF% static(18.6)
100	23.8	9.90 x10 ⁶	31.82	27.95
110	24.00	9.96 x10 ⁶	32.62	29.03
120	2403	10.19 x10 ⁶	35.68	30.64
130	24.7	10.27 x10 ⁶	36.75	32.79
140	25.1	10.61 x10 ⁶	41.27	34.94
150	25.8	10.78 x10 ⁶	43.54	38.70
160	26.3	10.81 x10⁶	43.94	41.39

Table: 3.5: Bending moment in 24.4 m steel bridge

Particulars	Value	Unit
Self weight of bridge @ 1730 kg/meter	16954	N/m
P. way load on bridge @ 0.4 tonnes/ meter	3920	N/m
Total dead load of bridge per meter	20874	N/m
Total BM due to dead load at mid span	1.71 x10⁶	N-m
Maxi. BM due to live load at mid span under static load condition	7.51 x10 ⁶	N-m
Maxi. BM due to live load at mid span under moving BOXN25 wagons	10.81x10⁶	N-m
Maxi. BM due to live load at mid span under moving Locos+BOXN25 wagons	1.4394x7.67x10 ⁶	N-m
	11.04x10⁶	
Total BM (DL+LL)	12.75 x10⁶	N-m

3.5 Stress at top and bottom flange of the bridge

Table: 5.6: Stresses in 24.4 m steel bridge

Particulars	Top flange	Bottom flange	Remark
Stress due to Live load	82.84 N/mm ²	82.84 N/mm ²	Ok
Stress due to Dead load	12.78 N/mm ²	12.78 N/mm ²	Ok
Total stress	95.62 N/mm ²	95.62 N/mm ²	Ok
Permissible stress	142.0 N/mm²	123.37N/mm²	Both stresses with in limit



3.6 Deflection criteria:

IRS Steel Bridge Code: Para 4.17: Deflection- - For permanent installation other than foot-over-bridges the ratio of deflection to length of the girder shall not exceed 1/600. In the case of foot over-bridges, the ratio of deflection to length of the girder shall not exceed 1/325.

Table: 5.7: Deflection in 24.4 m steel bridge

Particulars	value	Unit
Deflection due to dead load ($5wl^4/384EI$)	4.19	Mm
Deflection due to live load (static condition)	18.6	Mm
Deflection due to live load (BOXN25 wagon)	26.3	Mm
Deflection due to live load (Locos+BOXN25 wagon)	19.1x1.4139=27.01	Mm
Total deflection (DL+LL)	31.19	Mm
Total deflection (LL+IL) EUDL method	27.88	Mm
Permissible deflection (L/600)	42.66	Mm

3.7 CDA value:

It defines as the ratio of difference between dynamic response and static response of the bridge to static response of the bridge.

$$CDA = \frac{10.81 \times 10^6 - 7.51 \times 10^6}{7.51 \times 10^6}$$

CDA=43.94%

CDA as per the bridge rule APPENDIX-XXIII "25T loading-2088" for 25.6 m span

$$CDA = 0.15 + \frac{8}{6+L}$$

CDA=39.96%

Total load for 25.6 m effective span as per APPENDIX-XXIII "25T loading-2088" bridge rule =2537.96 KN

Static BM at mid span of the bridge =8.121x10⁶ N-m

Impact load on the bridge = 2537.96x0.3996 KN
=1014.17 KN

Total live load and impact load =3552.13 KN

BM at mid span of the bridge due to live load and impact load
=11.366x10⁶ N-m

BM at mid span of the bridge due to BOXN25 wagon vehicles =11.04x10⁶ N-m



5.8 Conclusion:

Dynamic response of Railway Bridge under moving railway vehicles has been observed to be depending on dynamic characteristics of the vehicle such as the axle load, sprung and unsprung masses, stiffness of primary and secondary suspension system, damping, vehicle speed, type of wagons, wagon dimension load combination and interaction between them. Throughout the study 24.4 m clear span bridge for “MBG loading-1987” bridge rule as per RDSO Drawing B/16005 is considered. In case no. 1 & 2, live load on bridge is considered as per load combination-1, “25t loading-2008” bridge rule. It is true; during the course of this study found that maximum responses of bridge occur when rear bogie of second loco and wagons are occupying the bridge. But dynamic analysis of the bridge in case no 3 carried out by considering the only a series of four BOXN25 wagons, locos are eliminated from load combination-1, “25t loading-2008” bridge rule, why this is because to make the interaction problem little bit simple. After this study, there are some observations.

Observation no.1

It is clear from table 3.4 that dynamic responses of bridge increased with increasing the speed of rolling stock over the bridge. Dynamic response found maxi under BOXN25 wagon if we compare in fig. 3.10, so BOXN wagon is critical from Dynamic response point of view.

Observation no.2

Found that midpoint displacement and bending moment at the center of bridge calculated by EUDL method as per bridge rule (“25t Loading-2008”) is slightly higher than the midpoint displacement and bending moment of bridge obtains by dynamic analysis of a series of four BOXN25 wagon, if in case of two locos with axle load 25 considered to the series of four BOXN25 Wagon, it may or may not be true.



Observation no.3

CDA value obtained by dynamic analysis is higher than the CDA obtained by conventional method as per bridge rule at speed 140 kmph and above.

Observation no.4

Stresses in the top and bottom flanges of the bridge are calculated as per dynamic analysis (only for DL+LL) and found that stresses are within permissible limit prescribed in RDSO Drawing B/16005 for 24.4 m clear span bridge “MBG loading 1987” specifically under the four BOXN25 wagon.

Observation no.5

Midpoint deflection of the bridge in case of dynamic analysis (DL+LL) is less than to deflection calculated by EUDL method (DL+LL+IL) both are within permissible limit. Specifically under the action of four BOXN25 wagon

Finally we can conclude, that for (DL+LL) (“25t Loading-2008”) bridge rule, load combination 1, bridge had satisfactory results under the action a series of four BOXN25 wagon. For further study in continuation, locos to be considered to get more exact dynamic response of Bridge and Vehicle

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