



# **Stress Assessment in Railway Foundation System for Semi High Speed and High Axle Loads Trains.**

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## **Abstract**

*A more precise comprehension of the dynamic behaviour of track and the track foundation is much needed exercise to be done when semi high speed and high load capacity trains are need of the hour. The complexities involved in the above said circumstance are puncturing of ballast into subgrade, uneven settlement of the track foundation and cant deficiency. This paper henceforth intends to examine the existing methodologies for assessment of track and its foundation stresses. Also emphasis has been laid to relate these methods to Indian railway scenario and critically examine whether the existing track structure is suitable for semi-high speed and high axle load formations with suggestions of what should be done to augment the same.*

## **1. Introduction:**

To increase the modal share in the freight sector, to improve the financial status and to keep Indian railways in vogue competing with other modes of transport- augmentation of freight carrying capacity and running trains at faster speeds is a much needed move. But this cannot be done until the new tracks are appropriately designed and existing track structure is critically analysed to access its capability to sustain the proposed changes. Most of the existing track has been designed before independence and specifications

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which better suited to the then demands. Though over the decades constant improvement has been done to track system like PSC sleepers, CMS crossing, LWR, mechanised track maintenance, better fasteners etc., to keep pace with increasing axle load and speed. Still it cannot be ascertained with surety that present track structure would sustain the higher impact loads and speeds until a detailed analysis is done both theoretically and experimentally.

## 2. **Analytical and Numerical Methods of Stress Determination:**

The challenges that would be faced in case of high axle load would be degradation of track foundation due to excess stresses in subgrade, puncturing of ballast into subgrade, uneven settlement of subgrade on curves etc. and in case of semi-high speed is cant.

A systematic study has to be done to understand the stresses that would develop in track foundation in order to address excess stresses in subgrade with effective subgrade properties. Also comprehension of settlements in ballast and subgrade is a must, as these settlements if uneven might further affect the cant deficiency. Hence this review paper is divided into three parts where in the first part work done by researchers so far in terms of stresses in normal foundations and railway track foundation would be discussed and in the later two parts the work done in arena of settlements and dynamic augmentation would be examined. The following work has been done by various researchers to access the stress in foundations.

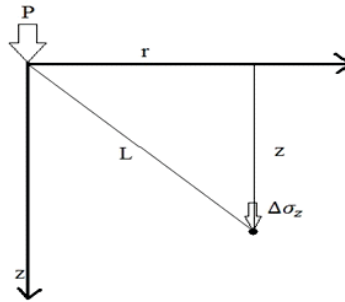
### 2.1 **Boussinesq Method:**

Equations for the stresses and strains induced in homogeneous, isotropic, weightless, linearly elastic half space, with a plane horizontal surface, by a point load perpendicular to the surface and acting at the surface, was first solved in usable form by Boussinesq (1885).

$$\Delta\sigma_z = \frac{3Pz^3}{2\pi L^5} \text{ where } L = (r^2 + z^2)^{\frac{5}{2}}$$



Where P is point load, z is depth, L is radial distance as seen in above figure. Apart from point load, formulas for line load, rectangular load and circular load have also been given. The drawback of this method is that the soil properties have been completely neglected.



**FIGURE 1 SCHEMATIC OF BOUSSINESQ METHOD**

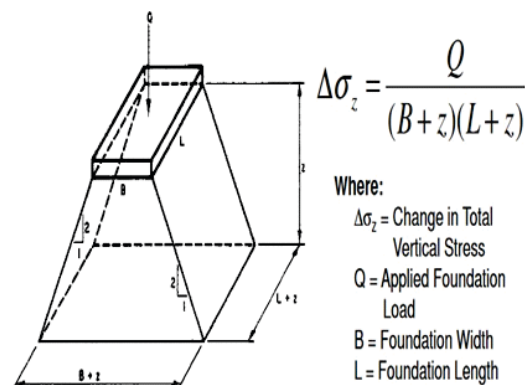
## 2.2 Westergaards Method:

A more judicious assumption has to be made by westergaard unlike boussinesq where he assumed soil to be homogenous and isotropic which is not. The soil according to westergaard can be assumed to be laterally reinforced by numerous sheets of negligible thickness but of infinite rigidity which prevent the mass as a whole from undergoing lateral movement of soil grains. The following formula has been proposed which includes the soil property i.e. poisons ratio.

$$\sigma_z = \frac{Q}{2\pi z^2} \frac{\sqrt{\frac{1-2\mu}{2-2\mu}}}{\left[\frac{1-2\mu}{2-\mu} + \left(\frac{r}{z}\right)^2\right]^{\frac{3}{2}}}$$

## 2.3 2H: 1V method–

One of the crudest approximate method to access the stress distribution which assumes that the total applied load on the surface of the soil is distributed over an area of the same shape as the loaded area on the surface, but with dimensions



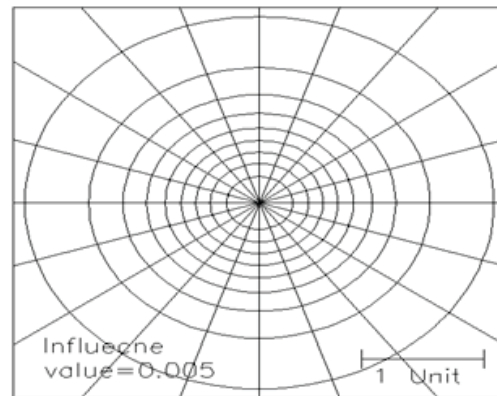
**FIGURE SCHEMATIC FOR 2H: 1V METHOD**



that increase by an amount equal to the horizontal to 1 vertical below the surface.

## 2.4 Newmark's Method:

Newmark has designed charts for finding the stresses under an irregularly loaded area which is not geometrically simple. The chart consists of influence areas bounded by radial lines and circular arcs. The loaded area under which stress is to be found is drawn to scale such that length of scale line on chart



**FIGURE 3** NEWMARK CHART

represents the depth at which stress is to be found. The position of the loaded area on chart is so placed such that the point where stress is to be determined is placed at the centre of the chart. The product of the number of influence areas (N), influence value of the chart ( $I_p$ ) and the stress on loaded surface would give the stress at required depth.

$$\sigma = N \times I_p \times q$$

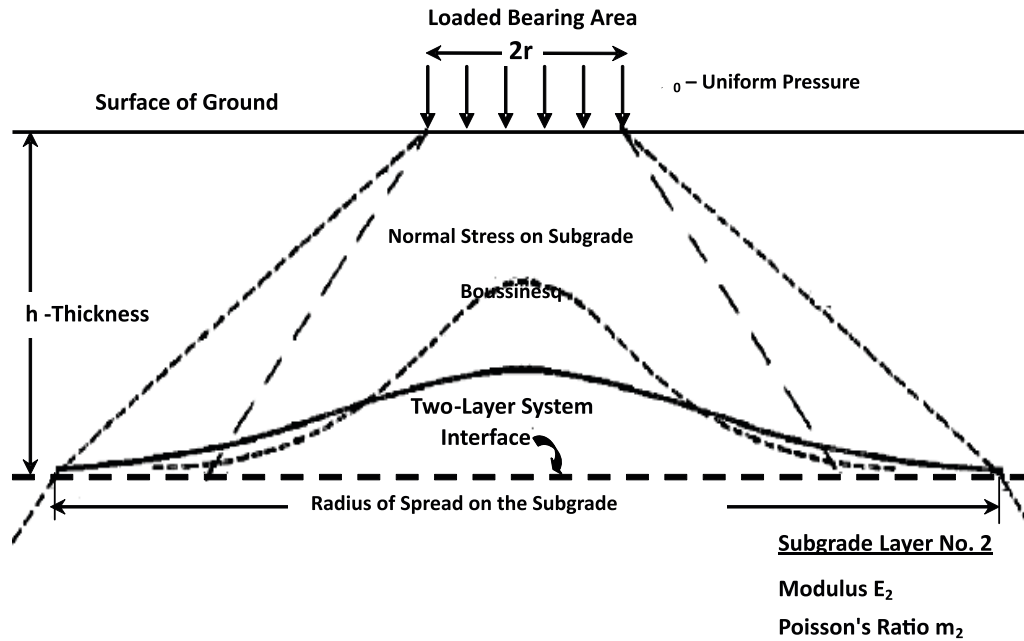
## 2.5 Burmister's Method

Stress and displacements within two and three layer system was studied by Burmister (1943) in his series of paper. In his paper, he derived the basic equations of stresses and settlements with the methods of the theory of elasticity. Some assumptions and certain essential boundary and continuity conditions across the interface between the two layers have been taken in his derivation.

In series (I) paper, he is assumed that the two layers of the system are continuously in contact with shearing resistance, so that the two layers act together as an full continuity of stress and displacement across the interface between the layers, developing the theory of the two-layer system, which were derived by Love to satisfy the equations of equilibrium and compatibility.



Two-layer system of layer No.1 and layer No.2 with moduli  $E_1$  and  $E_2$  is shown in Figure.



Equation of equilibrium

Figure 4 TWO LAYER SYSTEM

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad \frac{\partial \sigma_r}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0$$

Equation of compatibility

$$\nabla^4 \phi = 0$$

$$\nabla^2 = \left[ \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right]$$

Elasticity equation of stress

$$\sigma_z = \frac{\partial}{\partial z} \left[ (2-\mu) \nabla \phi - \frac{\partial^2 \phi}{\partial z^2} \right]$$

$$\sigma_r = \frac{\partial}{\partial z} \left[ \mu \nabla \phi - \frac{\partial^2 \phi}{\partial r^2} \right]$$

$$\sigma_\theta = \frac{\partial}{\partial z} \left[ \mu \nabla \phi - \frac{1}{r} \frac{\partial \phi}{\partial r} \right]$$

$$\sigma_{rz} = \frac{\partial}{\partial r} \left[ (1-\mu) \nabla \phi - \frac{\partial^2 \phi}{\partial z^2} \right]$$



## Elasticity equation of displacement

$$w = \frac{1+\mu}{E} \left[ (1-2\mu) \nabla^2 \phi + \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} \right]$$

$$u = -\frac{1+\mu}{E} \left[ \frac{\partial^2 \phi}{\partial r^2} \right]$$

## Equations for the two-layer system full continuity

The following form of stress function ( $\phi$ ) have been taken in term of Bessel functions and a parameter “m”

$$\text{Stress Function } \phi = J_0(mr) [Ae^{mz} - Be^{-mz} + Cze^{mz} - Dze^{-mz}]$$

$$\text{Settlement } w = J_0(mr) \frac{2(1-\mu_1^2)}{E_1} \left[ \frac{1 + 4Kmhe^{-2mh} - KLe^{-4mh}}{1 - (L + K + 4Km^2h^2)e^{-2mh} + KLe^{-4mh}} \right]$$

$$\text{Approximate radius of curvature of surface, } \frac{1}{R} = \frac{\partial^2 w}{\partial r^2}$$

$$\frac{1}{R} = - \left[ m^2 J_0(mr) - \frac{m^2 J_1(mr)}{mr} \right] \left[ \frac{1-\mu_1^2}{E_1} \right] \left[ \frac{1 + 4Kmhe^{-2mh} - KLe^{-4mh}}{1 - (L + K + 4Km^2h^2)e^{-2mh} + KLe^{-4mh}} \right]$$

## At the Interface between Layer No.1 and No.2

Normal stress

$$\sigma_z = -mJ_0(mr) \left[ \frac{1+mh-L/2-0.5K(1+2mh)e^{-mh} + [KL(1-mh)-L/2-0.5K(1-2mh)]e^{-3mh}}{1-(L+K+4Km^2h^2)e^{-2mh} + KLe^{-4mh}} \right]$$

Shearing stress

$$\tau_{rz} = -mJ_1(mr) \left[ \frac{[mh+L/2-0.5K(1+2mh)e^{-mh} + [KLmh-L/2+0.5K(1-2mh)]e^{-3mh}]}{1-(L+K+4Km^2h^2)e^{-2mh} + KLe^{-4mh}} \right]$$

Radial stress

$$\sigma_r = -mJ_0(mr) \left[ \frac{[1-mh+L/2-1.5K(1+2mh)]e^{-mh} + [KL(1+mh)+L/2-1.5K(1-2mh)]e^{-3mh}}{1-(L+K+4Km^2h^2)e^{-2mh} + KLe^{-4mh}} \right]$$

$$+ \frac{mJ_1(mr)}{mr} \left[ \frac{[1-2\mu_1-mh+L/2-0.5K(3-4\mu_1)(1+2mh)]e^{-mh} + [KL(1-2\mu_1+mh)+L/2-0.5K(3-4\mu_1)(1-2mh)]e^{-3mh}}{1-(L+K+4Km^2h^2)e^{-2mh} + KLe^{-4mh}} \right]$$



## Settlement

$$w = J_0(mr) \frac{1+\mu_1}{E_1} \left[ \frac{\left[ 2-2\mu_1+mh+L/2+0.5K(3-4\mu_1)(1+2mh) \right] e^{-mh} - \left[ KL(2-2\mu_1-mh)+L/2+0.5K(3-4\mu_1)(1-2mh) \right] e^{-3mh}}{1-(L+K+4Km^2h^2)e^{-2mh}+KLe^{-4mh}} \right]$$

Approximate radius of curvature of the interface

$$\frac{1}{R} = - \left[ m^2 J_0(mr) - \frac{m^2 J_1(mr)}{mr} \right] \left[ \frac{1+\mu_1^2}{E_1} \right] [\text{Bracket of Settlement Equation}]$$

Where the coefficient of the strength properties of the two layers are:

$$n = \frac{E_2}{E_1} \left[ \frac{1+\mu_1}{1+\mu_2} \right], \quad K = \left[ \frac{1-n}{1+n(3-4\mu_1)} \right], \quad L = \left[ \frac{(3-4\mu_2)-n(3-4\mu_1)}{(3-4\mu_2)+n} \right]$$

In series (II) paper, he is assumed that the two layers of the system are with a frictionless interface and same boundary conditions as discusses above, the equations of the theory of elasticity for the three-dimensional problem in cylindrical coordinates were employed, which were derived by Love to satisfy the equations of equilibrium and compatibility.

**At the Surface of Layer No.1**

## Settlement

$$w = J_0(mr) \frac{2(1-\mu_1^2)}{E_1} \left[ \frac{F - [2F-1-2mh]e^{-2h} - (1-F)e^{-4mh}}{F + [(2F-1)2mh - (1+2m^2h^2)]e^{-2mh} + (1-F)e^{-2mh}} \right]$$

Approximate radius of curvature,  $\frac{1}{R} = \frac{\partial^2 w}{\partial r^2}$

$$\frac{1}{R} = - \left[ m^2 J_0(mr) - \frac{m^2 J_1(mr)}{mr} \right] \left[ \frac{(1-\mu_1^2)}{E_1} \right] [\text{Bracket of Settlement Equation}] /$$



**At the Interface between Layer No.1 and No.2**

$$\text{Normal stress } \sigma_z = -mJ_0(mr) \left[ 2F - 1 \right] \left[ \frac{(1+mh)e^{-mh} - (1-mh)e^{-3mh}}{F + \left[ (2F-1)2mh - (1+2m^2h^2) \right] e^{-2mh} + (1-F)e^{-4mh}} \right]$$

Radial stress

$$\sigma_{r1} = mJ_0(mr) \left[ \frac{[(1+mh) - 2F(1-mh)]e^{-mh} - [(1+3mh) - 2F(1+mh)]e^{-3mh}}{F + \left[ (2F-1)2mh - (1+2m^2h^2) \right] e^{-2mh} + (1-F)e^{-4mh}} \right]$$

$$- \frac{mJ_1(mr)}{mr} \left[ \frac{[(1-2\mu_1)(1+mh) - 2F(1-2\mu_1-mh)]e^{-mh} - [(1-2\mu_1)(1+mh) + 2mh - 2F(1-2\mu_1+mh)]e^{-3mh}}{F + \left[ (2F-1)2mh - (1+2m^2h^2) \right] e^{-2mh} + (1-F)e^{-4mh}} \right]$$

$$\text{Settlement } w = J_0(mr) \frac{2(1-\mu_1^2)}{E_1} \left[ \frac{(1+mh)e^{-mh} - (1-mh)e^{-3mh}}{F + \left[ (2F-1)2mh - (1+2m^2h^2) \right] e^{-2mh} + (1-F)e^{-4mh}} \right]$$

**Approximate radius of curvature**

$$\frac{1}{R} = - \left[ m^2 J_0(mr) - \frac{m^2 J_1(mr)}{mr} \right] \left[ \frac{2(1-\mu_1^2)}{E_1} \right] [\text{Bracket of Settlement Equation}]$$

Where the coefficients of the strength properties of the two layers are:

$$n = \frac{E_2}{E_1} \left[ \frac{(1+\mu_1)}{(1+\mu_2)} \right], \quad F = \left[ \frac{(1-\mu_2) + n(1-\mu_1)}{2(1-\mu_2)} \right]$$

In series (III) paper, he is assumed that the three layers of the system are continuously in contact with shearing resistance fully active between them, so that the two layers act together as an full continuity of stress and displacement across the interface between the layers, developing the theory of the three-layer system, the equations of the theory of elasticity for the three-dimensional problem in cylindrical coordinates were employed, which were derived by Love to satisfy the equations of equilibrium and compatibility.





Three-Layer Settlement equation at the surface of ground

$$w = +J_0 (mr) \frac{2(1-\mu_1^2)}{E_1} \left[ \frac{\text{Numerator}}{\text{Denominator}} \right]$$

Numerator

$$\begin{aligned} & \left[ 1 + 4mHKe^{-2mH} - JKe^{-4mH} + 4mhNe^{-2mH}e^{-2mh} - LNe^{-4mH}e^{-4mh} \right. \\ & + \left[ 4mH \left( \frac{1-J}{1-K} \right) K^2 L + 16mHm^2 h^2 \left( \frac{1-J}{1-K} \right) K^2 N \right. \\ & \left. \left. + 4mH \left( \frac{1-K}{1-J} \right) N - 4mhJKN \right] e^{-2mH} e^{-2mh} \right. \\ & + \left[ \left( \frac{1-K}{1-J} \right) JN + \left( \frac{1-J}{1-K} \right) KL + 4m^2 h^2 \left( \frac{1-J}{1-K} \right) KN \right] \left[ e^{-2mh} - e^{-4mH} e^{-2mh} \right] \\ & \left. + 4mHKLNe^{-2mH} e^{-4mh} + JKLN e^{-4mh} \right] \end{aligned}$$

Denominator

$$\begin{aligned} & \left[ 1 - \left[ J + K + 4m^2 H^2 K \right] e^{-2mH} + JKe^{-4mH} \right. \\ & - \left[ \left( \frac{1-K}{1-J} \right) N + \left( \frac{1-J}{1-K} \right) L + 4m^2 h^2 \left( \frac{1-J}{1-K} \right) N \right] e^{-2mH} e^{-2mh} + LNe^{-4mH} e^{-4mh} \\ & - \left[ \left( 1 + 4m^2 H^2 \right) \left( \frac{1-J}{1-K} \right) K^2 L + 4m^2 h^2 \left( 1 + 4m^2 H^2 \right) \left( \frac{1-J}{1-K} \right) K^2 N \right. \\ & \left. + \left( \frac{1-K}{1-J} \right) J^2 N + 8mHmh(1-JK)N + 4m^2 H^2 \left( \frac{1-K}{1-J} \right) N \right] e^{-2mH} e^{-2mh} \\ & + \left[ \left( \frac{1-K}{1-J} \right) JN + \left( \frac{1-J}{1-K} \right) KL + 4m^2 h^2 \left( \frac{1-J}{1-K} \right) KN \right] \left[ e^{-2mh} + e^{-4mH} e^{-2mh} \right] \\ & \left. - \left[ \left( 1 + 4m^2 H^2 \right) KLN + JLN \right] e^{-2mH} e^{-4mh} + JKLN e^{-4mh} \right] \end{aligned}$$

Where the strength coefficients for the interface are as follows: “E” is the modulus and “m” is Passion's ratio.

$$N = \left[ \frac{1-n}{1+(3-4\mu_2)n} \right], L = \left[ \frac{(3-4\mu_3)-n(3-4\mu_2)}{(3-4\mu_3)+n} \right], n = \left[ \frac{E_3 \times \frac{1+\mu_2}{E_2}}{1+\mu_3} \right]$$

In addition, the strength coefficients for layers No.1 and No.2 are:

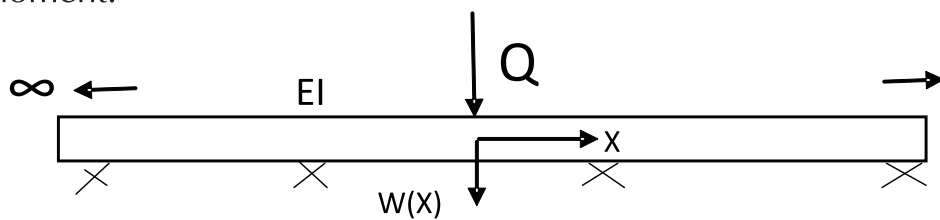
$$K = \left[ \frac{1-k}{1+(3-4\mu_1)k} \right], J = \left[ \frac{(3-4\mu_2)-(3-4\mu_1)k}{(3-4\mu_2)+k} \right], k = \left[ \frac{E_2 \times \frac{1+\mu_1}{E_1}}{1+\mu_2} \right]$$



In the further studied, he has presented the complete problem of stress and displacement in layer No.1 of a two-layer rigid base soil system. The two-layer rigid base soil system with a concentrated load  $P$ , applied at the surface of the ground. The characteristics of the distribution of stress throughout layer No.1 are discussed with particular reference to the favourable and unfavourable aspects and their implications in the stress investigations.

## 2.6 Methods suggested by Esvald:

All the above said methods have laid emphasis on calculation of stresses in normal foundations when load is applied. These are not directly applicable to railway embankments or track foundations. Very little amount of work has been done regarding stresses that develop in the rail, sleepers, ballast and subgrade in the event of dynamic load. Most of these works have been compiled in book by Esvald. Most of the pioneering work in terms of stresses in rail road has been done by Zimmermann and Eisenmann. Zimmermann has done longitudinal beam calculations on the track. It was assumed that winkler's hypothesis applies to track support. A beam of infinite length with unit width and bending stiffness  $EI$  continuously supported by an elastic foundation with foundation modulus  $C$  loaded by wheel load  $Q$  was solved to find the maximum bending moment.



**FIGURE 1 INFINITE BEAM MODEL**

$$EIW^{IV} + KW = 0$$

Upon solving the above equation displacement  $w(x)$  can be written as

$$w(x) = \frac{QL^3}{8EI} \vartheta(x) \quad ; \quad \vartheta(x) = e^{-\frac{x}{L}} \left( \cos \frac{x}{L} + \sin \frac{x}{L} \right)$$



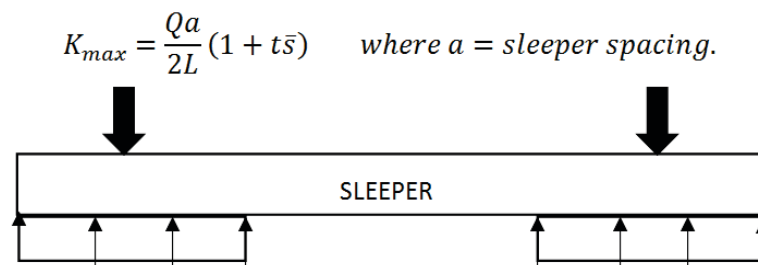
Where  $L$  = characteristic length =  $\sqrt[4]{\frac{4EI}{k}}$ ,  $k$  = foundation coefficient ( $N/m^2$ )

Based on these results it was concluded that maximum bending moment due to several wheel loads is significantly less than in case of single axle load. The only drawback of work done by Zimmermann lies in the fact that the dynamic effect of axle load was ignored which was rectified by Eisenmann. Based on the Zimmermann's work Eisenmann had included the dynamic effect and given an empirical formula for the calculation of bending tensile stresses in rail foot centre.

$$\sigma_{max} = \sigma_{mean}(1 + t\bar{s}), \quad \sigma_{mean} = \frac{QL}{4W}$$

in which  $W$  = section modulus, variation coefficient =  $0.1\phi$  for good tracks,  $0.3\phi$  for moderate tracks,  $\phi$  = speed increment =  $1 + \frac{V-60}{140}$  for  $v > 60$  Km/h,  $t$  = time increment = 3 recommended.

Considering the sleeper contact force to be uniformly distributed as shown in fig Eisenmann theory proposed maximum bearing force as



**Figure 6** Contact Stress Distribution On Sleeper

The sleeper spacing is found to be an highly influencing factor in terms of bearing force.

Further the stress on ballast bed and formation were also calculated by Eisenmann including dynamic. Maximum stress on ballast bed is given as

$$\sigma_{max} = \frac{Qa}{2LA}(1 + t\bar{s})$$

where  $A$  = contact area between sleeper and ballast bed for half sleeper



Magnitude of stress below various sleepers by wheel load Q is

$$\sigma_{max} = \frac{Qa}{2LA} (1 + ts) * e^{-\frac{x_i}{L}} \left( \cos \frac{x_i}{L} + \sin \frac{x_i}{L} \right); x_i \geq 0$$

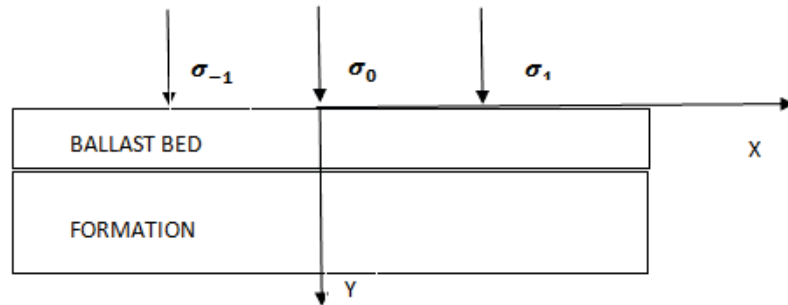


FIGURE 7 STRESS ON BALLAST BED AND FORMATION

### 3.0 Method of displacement/settlement determination

#### 3.1 Indraratna et al. (2004):

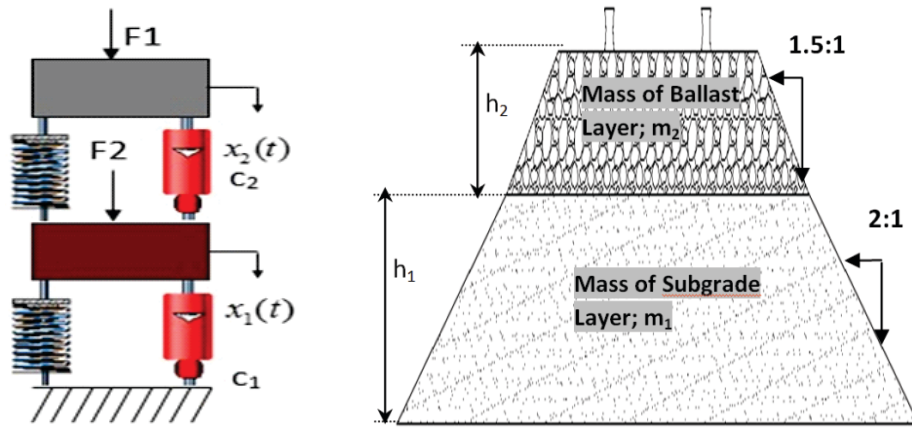
Indraratna et al. (2004) studied on ballast characteristics and the effect of geosynthetics on rail track deformation. The geotechnical properties of ballast that affect the performance of rail tracks were discussed. Main factors that affect ballast deformation such as confining pressure and particle size distribution were examined. The experimental results clearly showed that with insertion of any type of selected geosynthetics the extent of degradation and settlement in fresh and recycled ballast were reduced. Nonetheless, it is advocated to employ a bonded geosynthetic (geosynthetic to prevent something to pass-through) because of the need for preventing the ingress of liquefied mud into ballast voids under cyclic load, as well as to maintain efficient pore pressure dissipation.

#### 3.2 Indraratna et al. (2005):

Indraratna et al. (2005) Studied on the role of confining pressure on ballast degradation, and to evaluate whether there is an optimum confining pressure in the track to reduce particle breakage. The numbers of drained cyclic triaxial tests have been conducted on Latte Basalt, which is a ballast commonly used on Sydney-Wollongong track.

### 3.3 Choudhury et al (2008):

A two degree of freedom mass spring dashpot system was used to model the track foundation system and was found to be simple and sufficient to evaluate the settlement of the track foundation comparing to realistic values. The ballast was assumed to be an elastic layer and the subgrade as an elastic half space to make the 2-DOF approach more realistic.



**FIGURE 8** SCHEMATIC OF 2DOF MODEL

To validate the analytical model, previously published experimental and analytical data (e.g. Bowles 1996) with respect to stiffness, damping ratio and other geotechnical properties for both the ballast and the subgrade were used. The load distribution was assumed to be uniform at the interface of the ballast and the subgrade layers, and the interface was considered purely frictional in nature, i.e. no slip condition.

The following differential equations were derived by considering the dynamic equilibrium of the two masses  $m_1$  and  $m_2$  using an individual free body diagram and D'Alembert's principle

$$\left. \begin{aligned} m_1 \ddot{x}_1(h_1, t) + c_1 \dot{x}_1(h_1, t) + c_2 [\dot{x}_1(h_1, t) - \dot{x}_2(h_2, t)] + k_1 x_1(h_1, t) \\ - k_2 [x_1(h_1, t) - x_2(h_2, t)] = f_1(t) \end{aligned} \right\} \text{Eq(1)}$$

$$m_2 \ddot{x}_2(h_2, t) + c_2 [\dot{x}_2(h_2, t) - \dot{x}_1(h_1, t)] + k_2 [x_2(h_2, t) - x_1(h_1, t)] = f_2(t) \quad \text{Eq(2)}$$

Where  $\ddot{x}_1(h_1, t)$ ,  $\dot{x}_1(h_1, t)$ ,  $x_1(h_1, t)$  and  $\ddot{x}_2(h_2, t)$ ,  $\dot{x}_2(h_2, t)$ ,  $x_2(h_2, t)$  are the vertical acceleration,



velocity and displacement for the masses ' $m_1$ ' and ' $m_2$ ' respectively. The thickness of the ballast layer is ' $h_1$ ' and that of subgrade layer is ' $h_2$ '. In matrix form, Equations (1) and (2) can be written as

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix}$$

The above system of equations was solved by using Newmark's integration method and the displacements i.e. settlements were found. This method by far has proven to be one of the simplest and effective method to calculate the settlements in ballast and subgrade. The elastic settlements for various subgrades have been calculated as shown in figure 6 and were matching with realistic values.

Type of soil	Maximum Displacement (mm)	
	Subgrade	Ballast
Dense Uniform Sand	3.2	3.1
Loose Uniform Sand	5.6	5.4
Stiff Clay	6.5	6.4
Soft Clay	9.2	9.1

**Figure 9** Maximum Displacement Of Subgrade And Ballast Layer Due To Cyclic Loading

#### 4. Dynamic augmentation factor:

Train axle loads are dynamic in nature, hence considering static loads and finding stresses would not yield realistic results. But it is complex to do dynamic analysis and find the stresses, hence an impact factor is used to amplify the static load to arrive at near correct values of stresses calculated when a dynamic analysis is done. Various parameters have been used by researches to arrive at dynamic impact factor presented in the table below.

Dynamic factor	Expression	Vehicle parameter		Track parameter
		Train speed	Wheel diameter	Track modulus
Talbot	$1 + \frac{33V}{100D}$	●	●	



Dynamic factor	Expression	Vehicle parameter		Track parameter
		Train speed	Wheel diameter	Track modulus
Talbot	$1 + \frac{33V}{100D}$	●	●	
Indian Railways	$1 + \frac{V}{3\sqrt{U}}$	●		●
German Railways		●		
Clarke	$1 + \frac{15V}{D\sqrt{U}}$	●		
WMATA	$(1+0.001V^2)^{2/3}$	●		

Train Speed (in mph)	Dynamic Impact factor				
	Talbot	Indian Railways	German Railways	Clarke	WMATA
30	1.275	1.129099445	1.0880146	1.161374	1.059134217
40	1.36667	1.172132593	1.1464672	1.215166	1.104007421
50	1.45833	1.215165741	1.213225	1.268957	1.160397208
60	1.55	1.25819889	1.2845368	1.322749	1.227512544
70	1.64167	1.301232038	1.3566514	1.37654	1.304540337
80	1.73333	1.344265186	1.4258176	1.430331	1.390686478
90	1.825	1.387298335	1.4882842	1.484123	1.485202654
100	1.91667	1.430331483	1.5403	1.537914	1.587401052
110	2.00833	1.473364631	1.5781138	1.591706	1.696660109
120	2.1	1.516397779	1.5979744	1.645497	1.812424333

It can evidently be seen that dynamic augmentation factor depends on various parameters and is sensitive to them. It is also observed that with increasing speed the dynamic load increases, which is almost double at 120Kmph. Hence this becomes crucial in the context of semi-high speed trains in Indian scenario.

## 5. Discussions

In the context of semi-high speed and high axle load two questions are to be addressed whether we have suitable design methods for new tracks and whether the existing track can meet the growing demands. An attempt has been made to answer these questions in this paper and it was found that very little amount of work has been





done so far. Hence the design of new lines for handling semi-high speed and high axle load trains must be done only after a proper conceptual comprehension and experimental work is done. The following exercises are to be done before we augment on speed and load

1. Experimental work has to be done to observe the effect of formation improvement techniques.
2. The effect of high speed in terms of dynamic augmentation has to be analysed as the static load gets nearly doubled as speed increases. It has to be ascertained whether the existing track is capable of handling these load increments.
3. Analytical models have to be developed which better reciprocate the realistic track and foundation to critically examine its behaviour under high speed and heavy axle loads.
4. The settlements of the track foundation is a grey area where work has to be done. In case of cant deficiency the force on outer rail is more thereby causing excessive settlement under outer rail when compared to inner rail. This causes pseudo cant deficiency which is undesirable.
5. The stresses in subgrade must be established to improve upon its characteristics using some soil improvement techniques like geosynthetics.

## **6. Conclusions:**

With ever growing demands the need of semi-high speed and high axle trains is a much needed step to be taken. But before this a careful analysis and caution should be exercised to design the new tracks and to improve the existing tracks. The increasing speed would lead to increase in dynamic effect which at higher speeds might cause the dynamic load equal to twice the static load. Adding to it if the static load in itself is increased in case of high axle load, the dynamic effect at high speeds would be very high. Hence alongside the design of new track, augmenting of existing track in itself is a big challenge to be tackled. With the comprehensive





review done in this paper it is understood that very little amount of research has been done in this area. Hence research work must be done in this regard which have been discussed above before we step into the new era of high speed and high axle loads.

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